► Types of Math Questions

Math questions on the ACT are classified by both *topic* and *skill level*. As noted earlier, the six general topics covered are:

Pre-Algebra
Elementary Algebra
Intermediate Algebra

Tips

- The math questions start easy and get harder. Pace yourself accordingly.
- Study wisely. The number of questions involving various algebra topics is significantly higher than the number of trigonometry questions. Spend more time studying algebra concepts.
- There is no penalty for wrong answers. Make sure that you answer all of the questions, even if some answers are only a guess.
- If you are not sure of an answer, take your best guess. Try to eliminate a couple of the answer choices.
- If you skip a question, leave that question blank on the answer sheet and return to it when you are done. Often, a question later in the test will spark your memory about the answer to a question that you skipped.
- Read carefully! Make sure you understand what the question is asking.
- Use your calculator wisely. Many questions are answered more quickly and easily without a calculator.
- Most calculators are allowed on the test. However, there are some exceptions. Check the ACT website (ACT.org) for specific models that are not allowed.
- Keep your work organized. Number your work on your scratch paper so that you can refer back to it while checking your answers.
- Look for easy solutions to difficult problems. For example, the answer to a problem that can be solved using a complicated algebraic procedure may also be found by "plugging" the answer choices into the problem.
- Know basic formulas such as the formulas for area of triangles, rectangles, and circles. The Pythagorean theorem and basic trigonometric functions and identities are also useful, and not that complicated to remember.

Coordinate Geometry
Plane Geometry
Trigonometry

In addition to these six topics, there are three skill levels: basic, application, and analysis. Basic problems require simple knowledge of a topic and usually only take a few steps to solve. Application problems require knowledge of a few topics to complete the problem. Analysis problems require the use of several topics to complete a multi-step problem.

The questions appear in order of difficulty on the test, but topics are mixed together throughout the test.

Pre-Algebra

Topics in this section include many concepts you may have learned in middle or elementary school, such as operations on whole numbers, fractions, decimals, and integers; positive powers and square roots; absolute value; factors and multiples; ratio, proportion, and percent; linear equations; simple probability; using charts, tables, and graphs; and mean, median, mode, and range.

Numbers

- **Whole numbers** Whole numbers are also known as counting numbers: 0, 1, 2, 3, 4, 5, 6, . . .
- **Integers** Integers are both positive and negative whole numbers including zero: . . . –3, –2, –1, 0, 1, 2, 3 . . .
- **Rational numbers** Rational numbers are all numbers that can be written as fractions ($\frac{2}{3}$), terminating decimals (.75), and repeating decimals (.666 . . .)
- **Irrational numbers** Irrational numbers are numbers that cannot be expressed as terminating or repeating decimals: π or $\sqrt{2}$.

Order of Operations

Most people remember the order of operations by using a mnemonic device such as PEMDAS or ***P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally*. These stand for the order in which operations are done:

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Multiplication and division are done in the order that they appear from left to right. Addition and subtraction work the same way—left to right.

Parentheses also include any grouping symbol such as brackets [ ], braces { }, or the division bar.

Examples

1. $-5 + 2 \times 8$

2. $9 + (6 + 2 \times 4) - 3^2$

Solutions

1. $-5 + 2 \times 8$
$-5 + 16$
11

2. $9 + (6 + 2 \times 4) - 3^2$

$9 + (6 + 8) - 3^2$

$9 + 14 - 9$

$23 - 9$

14

FRACTIONS

Addition of Fractions

To add fractions, they must have a common denominator. The common denominator is a common multiple of the denominators. Usually, the least common multiple is used.

Example

$\frac{1}{3} + \frac{2}{7}$	The least common denominator for 3 and 7 is 21.
$(\frac{1}{3} \times \frac{7}{7}) + (\frac{2}{7} \times \frac{3}{3})$	Multiply the numerator and denominator of each fraction by the same number so that the denominator of each fraction is 21.
$\frac{2}{21} + \frac{6}{21} = \frac{8}{21}$	Add the numerators and keep the denominators the same. Simplify the answer if necessary.

Subtraction of Fractions

Use the same method for multiplying fractions, except subtract the numerators.

Multiplication of Fractions

Multiply numerators and multiply denominators. Simplify the answer if necessary.

Example

$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

Division of Fractions

Take the reciprocal of (flip) the second fraction and multiply.

$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$

Examples

1. $\frac{1}{3} + \frac{2}{5}$

2. $\frac{9}{10} - \frac{3}{4}$

3. $\frac{4}{5} \times \frac{7}{8}$

4. $\frac{3}{4} \div \frac{6}{7}$

Solutions

1. $\frac{1 \times 5}{3 \times 5} + \frac{2 \times 3}{5 \times 3}$

$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

2. $\frac{9 \times 2}{10 \times 2} - \frac{3 \times 5}{4 \times 5}$

$\frac{18}{20} - \frac{15}{20} = \frac{3}{20}$

3. $\frac{4}{5} \times \frac{7}{8} = \frac{28}{40} = \frac{7}{10}$

4. $\frac{3}{4} \times \frac{7}{6} = \frac{21}{24} = \frac{7}{8}$

Exponents and Square Roots

An exponent tells you how many times to the base is used as factor. Any base to the power of zero is one.

Example

$14^0 = 1$

$5^3 = 5 \times 5 \times 5 = 125$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$11^2 = 11 \times 11 = 121$

Make sure you know how to work with exponents on the calculator that you bring to the test. Most scientific calculators have a y^x or x^y button that is used to quickly calculate powers.

When finding a square root, you are looking for the number that when multiplied by itself gives you the number under the square root symbol.

$\sqrt{25} = 5$

$\sqrt{64} = 8$

$\sqrt{169} = 13$

Have the perfect squares of numbers from 1 to 13 memorized since they frequently come up in all types of math problems. The perfect squares (in order) are:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169.

Absolute Value

The absolute value is the distance of a number from zero. For example, $|-5|$ is 5 because -5 is 5 spaces from zero. Most people simply remember that the absolute value of a number is its positive form.

$|-39| = 39$
$|92| = 92$
$|-11| = 11$
$|987| = 987$

Factors and Multiples

Factors are numbers that divide evenly into another number. For example, 3 is a factor of 12 because it divides evenly into 12 four times.

6 is a factor of 66
9 is a factor of 27
-2 is a factor of 98

Multiples are numbers that result from multiplying a given number by another number. For example, 12 is a multiple of 3 because 12 is the result when 3 is multiplied by 4.

66 is a multiple of 6
27 is a multiple of 9
98 is a multiple of -2

Ratio, Proportion, and Percent

Ratios are used to compare two numbers and can be written three ways. The ratio 7 to 8 can be written 7:8, $\frac{7}{8}$, or in the words "7 to 8."

Proportions are written in the form $\frac{2}{5} = \frac{x}{25}$. Proportions are generally solved by cross-multiplying (multiply diagonally and set the cross-products equal to each other). For example,

$\frac{2}{5} = \frac{x}{25}$
$(2)(25) = 5x$
$50 = 5x$
$10 = x$

Percents are always “out of 100.” 45% means 45 out of 100. It is important to be able to write percents as decimals. This is done by moving the decimal point two places to the left.

45% = 0.45
3% = 0.03
124% = 1.24
0.9% = 0.009

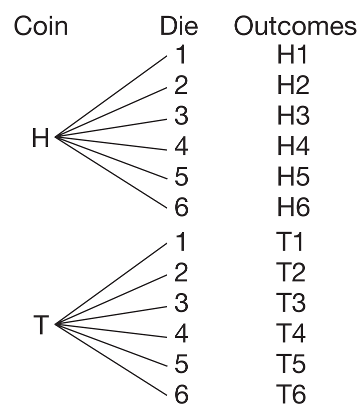
Probability

The probability of an event is $P(event) = \frac{\textit{favorable}}{\ }$.

For example, the probability of rolling a 5 when rolling a 6-sided die is $\frac{1}{6}$, because there is one favorable outcome (rolling a 5) and there are 6 possible outcomes (rolling a 1, 2, 3, 4, 5, or 6). If an event is impossible, it cannot happen, the probability is 0. If an event definitely will happen, the probability is 1.

Counting Principle and Tree Diagrams

The *sample space* is a list of all possible outcomes. A *tree diagram* is a convenient way of showing the sample space. Below is a tree diagram representing the sample space when a coin is tossed and a die is rolled.



The first column shows that there are two possible outcomes when a coin is tossed, either heads or tails. The second column shows that once the coin is tossed, there are six possible outcomes when the die is rolled, numbers 1 through 6. The outcomes listed indicate that the possible outcomes are: getting a heads, then rolling a 1; getting a heads, then rolling a 2; getting a heads, then rolling a 3; etc. This method allows you to clearly see all possible outcomes.

Another method to find the number of possible outcomes is to use the *counting principle*. An example of this method is on the following page.

Nancy has 4 pairs of shoes, 5 pairs of pants, and 6 shirts. How many different outfits can she make with these clothes?

Shoes	Pants	Shirts
4 choices	5 choices	6 choices

To find the number of possible outfits, multiply the number of choices for each item.

$4 \times 5 \times 6 = 120$

She can make 120 different outfits.

Helpful Hints about Probability

- If an event is certain to occur, the probability is 1.
- If an event is certain NOT to occur, the probability is 0.
- If you know the probability of all other events occurring, you can find the probability of the remaining event by adding the known probabilities together and subtracting that sum from 1.

Mean, Median, Mode, and Range

Mean is the average. To find the mean, add up all the numbers and divide by the number of items.

Median is the middle. To find the median, place all the numbers in order from least to greatest. Count to find the middle number in this list. Note that when there is an even number of numbers, there will be two middle numbers. To find the median, find the average of these two numbers.

Mode is the most frequent or the number that shows up the most. If there is no number that appears more than once, there is no mode.

The range is the difference between the highest and lowest number.

Example

Using the data 4, 6, 7, 7, 8, 9, 13, find the mean, median, mode, and range.

Mean: The sum of the numbers is 54. Since there are seven numbers, divide by 7 to find the mean. $54 \div 7 = 7.71$.

Median: The data is already in order from least to greatest, so simply find the middle number. 7 is the middle number.

Mode: 7 appears the most often and is the mode.

Range: $13 - 4 = 9$.

Linear Equations

An equation is solved by finding a number that is equal to an unknown variable.

Simple Rules for Working with Equations

1. The equal sign separates an equation into two sides.
2. Whenever an operation is performed on one side, the same operation must be performed on the other side.
3. Your first goal is to get all of the variables on one side and all of the numbers on the other.
4. The final step often will be to divide each side by the coefficient, leaving the variable equal to a number.

Cross-Multiplying

You can solve an equation that sets one fraction equal to another by **cross-multiplication**. Cross-multiplication involves setting the products of opposite pairs of terms equal.

Example

$\frac{x}{6} = \frac{x+10}{12}$ becomes $12x = 6(x) + 6(10)$

$12x = 6x + 60$

$-6x \qquad -6x$

$\frac{6x}{6} = \frac{60}{6}$

Thus, $x = 10$

Checking Equations

To check an equation, substitute the number equal to the variable in the original equation.

Example

To check the equation from the previous page, substitute the number 10 for the variable x.

$\frac{x}{6} = \frac{x+10}{12}$

$\frac{10}{6} = \frac{10+10}{12}$

$\frac{10}{6} = \frac{20}{12}$

Simplify the fraction on the right by dividing the numerator and denominator by 2.

$\frac{10}{6} = \frac{10}{6}$

Because this statement is true, you know the answer $x = 10$ is correct.

Special Tips for Checking Equations

1. If time permits, be sure to check all equations.

2. Be careful to answer the question that is being asked. Sometimes, this involves solving for a variable and than performing an operation.

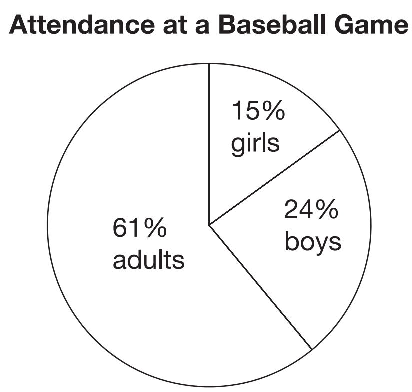
Example: If the question asks for the value of $x - 2$, and you find $x = 2$, the answer is not 2, but $2 - 2$. Thus, the answer is 0.

CHARTS, TABLES, AND GRAPHS

The ACT Math Test will assess your ability to analyze graphs and tables. It is important to read each graph or table very carefully before reading the question. This will help you to process the information that is presented. It is extremely important to read all of the information presented, paying special attention to headings and units of measure. Here is an overview of the types of graphs you will encounter:

- CIRCLE GRAPHS or PIE CHARTS

  This type of graph is representative of a whole and is usually divided into percentages. Each section of the chart represents a portion of the whole, and all of these sections added together will equal 100% of the whole.



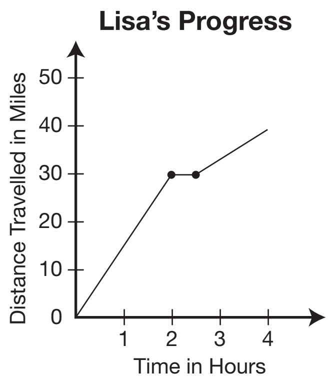
- BAR GRAPHS

  Bar graphs compare similar things with bars of different length, representing different values. These graphs may contain differently shaded bars used to represent different elements. Therefore, it is important to pay attention to both the size and shading of the graph.

- BROKEN LINE GRAPHS

Broken-line graphs illustrate a measurable change over time. If a line is slanted up, it represents an increase, whereas a line sloping down represents a decrease. A flat line indicates no change.

In the line graph below, Lisa's progress riding her bike is graphed. From 0 to 2 hours, Lisa moves steadily. Between 2 and $2\frac{1}{2}$ hours, Lisa stops (flat line). After her break, she continues again but at a slower pace (line is not as steep as from 0 to 2 hours).



Elementary Algebra

Elementary algebra covers many topics typically covered in an Algebra I course. Topics include operations on polynomials; solving quadratic equations by factoring; linear inequalities; properties of exponents and square roots; using variables to express relationships; and substitution.

Operations on Polynomials

Combining Like Terms: terms with the same variable and exponent can be combined by adding the coefficients and keeping the variable portion the same.

For example,

$4x^2 + 2x - 5 + 3x^2 - 9x + 10 =$
$7x^2 - 7x + 5$

Distributive Property: multiply all the terms inside the parentheses by the term outside the parentheses.

$7(2x - 1) = 14x - 7$

Solving Quadratic Equations by Factoring

Before factoring a quadratic equation to solve for the variable, you must set the equation equal to zero.

$x^2 - 7x = 30$
$x^2 - 7x - 30 = 0$

Next, factor.

$(x + 3)(x - 10) = 0$

Set each factor equal to zero and solve.

$x + 3 = 0 \qquad x - 10 = 0$

$x = -3 \qquad\quad x = 10$

The solution set for the equation is $\{-3, 10\}$.

Solving Inequalities

Solving inequalities is the same as solving regular equations, with one exception. The exception is that when multiplying or dividing by a negative, you must change the inequality symbol.

For example,

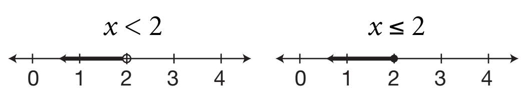
$-3x < 9$

$\frac{-3x}{-3} < \frac{9}{-3}$

$x > -3$

Notice that the inequality switched from *less than* to *greater than* after division by a negative.

When graphing inequalities on a number line, recall that < and > use open dots and ≤ and ≥ use solid dots.



Properties of Exponents

When multiplying, add exponents.

$x^3 \cdot x^5 = x^{3+5} = x^8$

When dividing, subtract exponents.

$\frac{x^7}{x^2} = x^{7-2} = x^5$

When calculating a power to a power, multiply.

$(x^6)^3 = x^{6 \cdot 3} = x^{18}$

Any number (or variable) to the zero power is 1.

$5^0 = 1$ $m^0 = 1$ $9{,}837{,}475^0 = 1$

Any number (or variable) to the first power is itself.

$5^1 = 5$ $m^1 = m$ $9{,}837{,}475^1 = 9{,}837{,}475$

Roots

Recall that exponents can be used to write roots. For example, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[3]{x} = x^{\frac{1}{3}}$. The denominator is the root. The numerator indicates the power. For example, $(\sqrt[3]{x})^4 = x^{\frac{4}{3}}$ and $\sqrt{x^5} = x^{\frac{5}{2}}$. The properties of exponents outlined above apply to fractional exponents as well.

Using Variables to Express Relationships

The most important skill needed for word problems is being able to use variables to express relationships. The following will assist you in this by giving you some common examples of English phrases and their mathematical equivalents.

- "Increase" means add.
  Example
  A number increased by five = $x + 5$.
- "Less than" means subtract.
  Example
  10 less than a number = $x - 10$.
- "Times" or "product" means multiply.
  Example
  Three times a number = $3x$.
- "Times the sum" means to multiply a number by a quantity.
  Example
  Five times the sum of a number and three = $5(x + 3)$.
- Two variables are sometimes used together.
  Example
  A number y exceeds five times a number x by ten.
  $y = 5x + 10$
- Inequality signs are used for "at least" and "at most," as well as "less than" and "more than."
  Examples
  The product of x and 6 is greater than 2.
  $x \times 6 > 2$

When 14 is added to a number x, the sum is less than 21.

$x + 14 < 21$

The sum of a number x and four is at least nine.

$x + 4 \geq 9$

When seven is subtracted from a number x, the difference is at most four.

$x - 7 \leq 4$

Assigning Variables in Word Problems

It may be necessary to create and assign variables in a word problem. To do this, first identify an unknown and a known. You may not actually know the exact value of the "known," but you will know at least something about its value.

Examples

Max is three years older than Ricky.
Unknown = Ricky's age = x
Known = Max's age is three years older
Therefore,
Ricky's age = x and Max's age = $x + 3$

Siobhan made twice as many cookies as Rebecca.
Unknown = number of cookies Rebecca made = x
Known = number of cookies Siobhan made = $2x$

Cordelia has five more than three times the number of books that Becky has.
Unknown = the number of books Becky has = x
Known = the number of books Cordelia has = $3x + 5$

Substitution

When asked to substitute a value for a variable, replace the variable with the value.

Example

Find the value of $x^2 + 4x - 1$, for $x = 3$.

Replace each x in the expression with the number 3. Then, simplify.

$= (3)^2 + 4(3) - 1$
$= 9 + 12 - 1$
$= 20$

The answer is 20.

Intermediate Algebra

Intermediate algebra covers many topics typically covered in an Algebra II course such as the quadratic formula; inequalities; absolute value equations; systems of equations; matrices; functions; quadratic inequalities; radical and rational expressions; complex numbers; and sequences.

The Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for quadratic equations in the form $ax^2 + bx + c = 0$.

The quadratic formula can be used to solve any quadratic equation. It is most useful for equations that cannot be solved by factoring.

Absolute Value Equations

Recall that both $|5| = 5$ and $|-5| = 5$. This concept must be used when solving equations where the variable is in the absolute value symbol.

$|x + 4| = 9$

$x + 4 = 9$ or $x + 4 = -9$

$x = 5$ $\quad$ $x = -13$

Systems of Equations

When solving a system of two linear equations with two variables, you are looking for the point on the coordinate plane at which the graphs of the two equations intersect. The elimination or addition method is usually the easiest way to find this point.

Solve the following system of equations:

$y = x + 2$

$2x + y = 17$

First, arrange the two equations so that they are both in the form $Ax + By = C$.

$-x + y = 2$

$2x + y = 17$

Next, multiply one of the equations so that the coefficient of one variable (we will use y) is the opposite of the coefficient of the same variable in the other equation.

$$\begin{aligned} -1(-x + y &= 2) \\ 2x + y &= 17 \\ \hline x - y &= -2 \\ 2x + y &= 17 \end{aligned}$$

Add the equations. One of the variables should cancel out.

$3x = 15$

Solve for the first variable.

$x = 5$

Find the value of the other variable by substituting this value into either original equation to find the other variable.

$y = 5 + 2$
$y = 7$

Since the answer is a point on the coordinate plane, write the answer as an ordered pair.

$(5, 7)$

Complex Numbers

Any number in the form $a + bi$ is a complex number. $i = \sqrt{-1}$. Operations with i are the same as with any variable, but you must remember the following rules involving exponents.

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

This pattern repeats every fourth exponent.

Rational Expressions

Algebraic fractions (rational expressions) are very similar to fractions in arithmetic.

Example

Write $\frac{x}{5} - \frac{x}{10}$ as a single fraction.

Solution

Just like in arithmetic, you need to find the lowest common denominator (LCD) of 5 and 10, which is 10. Then change each fraction into an equivalent fraction that has 10 as a denominator.

$$\frac{x}{5} - \frac{x}{10} = \frac{x(2)}{5(2)} - \frac{x}{10}$$
$$= \frac{2x}{10} - \frac{x}{10}$$
$$= \frac{x}{10}$$

Radical Expressions

- Radicals with the same radicand (number under the radical symbol) can be combined the same way "like terms" are combined.

Example

$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

Think of this as similar to:

$2x + 5x = 7x$

- To multiply radical expressions with the same root, multiply the radicands and simplify.

Example

$\sqrt{3} \cdot \sqrt{6} = \sqrt{18}$

This can be simplified by breaking 18 into 9 × 2.

$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

- Radicals can also be written in exponential form.

Example

$\sqrt[3]{x^5} = x^{\frac{5}{3}}$

In the fractional exponent, the numerator (top) is the power and the denominator (bottom) is the root.

By representing radical expressions using exponents, you are able to use the rules of exponents to simplify the expression.

Inequalities

The basic solution of linear inequalities was covered in the Elementary Algebra section. Following are some more advanced types of inequalities.

Solving Combined (or Compound) Inequalities

To solve an inequality that has the form $c < ax + b < d$, isolate the letter by performing the same operation on each member of the equation.

Example

If $-10 < -5y - 5 < 15$, find y.

Add five to each member of the inequality.

$-10 + 5 < -5y - 5 + 5 < 15 + 5$

$-5 < -5y < 20$

Divide each term by –5, changing the direction of both inequality symbols:

$\frac{-5}{-5} < \frac{-5y}{-5} < \frac{20}{-5} = 1 > y > -4$

The solution consists of all real numbers less than 1 and greater than –4.

Absolute Value Inequalities

$|x| < a$ is equivalent to $-a < x < a$ and $|x| > a$ is equivalent to $x > a$ or $x < -a$

Example

$|x + 3| > 7$

$x + 3 > 7$ or $x + 3 < -7$

$x > 4$ $\quad$ $x < -10$

Thus, $x > 4$ or $x < -10$.

Quadratic Inequalities

Recall that quadratic equations are equations of the form $ax^2 + bx + c = 0$.

To solve a quadratic inequality, first treat it like a quadratic equation and solve by setting the equation equal to zero and factoring. Next, plot these two points on a number line. This divides the number line into three regions. Choose a test number in each of the three regions and determine the sign of the equation when it is the value of x. Determine which of the three regions makes the inequality true. This region is the answer.

Example

$x^2 + x < 6$

Set the inequality equal to zero.

$x^2 + x - 6 < 0$

Factor the left side.

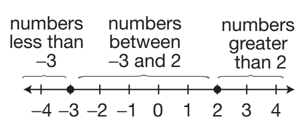
$(x + 3)(x - 2) < 0$

Set each of the factors equal to zero and solve.

$x + 3 = 0$ $\quad$ $x - 2 = 0$

$x = -3$ $\quad$ $x = 2$

Plot the numbers on a number line. This divides the number line into three regions.



The number line is divided into the following regions.

numbers less than –3

numbers between –3 and 2

numbers greater than 2

Use a test number in each region to see if $(x + 3)(x - 2)$ is positive or negative in that region.

numbers less than –3	*numbers between –3 and 2*	*numbers greater than 2*
test # = –5	test # = 0	test # = 3
$(-5 + 3)(-5 - 2) = 14$	$(0 + 3)(0 - 2) = -6$	$(3 + 3)(3 - 2) = 6$
positive	negative	positive

The original inequality was $(x + 3)(x - 2) < 0$. If a number is less than zero, it is negative. The only region that is negative is between –3 and 2; $-3 < x < 2$ is the solution.

Functions

Functions are often written in the form $f(x) = 5x - 1$. You might be asked to find $f(3)$, in which case you substitute 3 in for x. $f(3) = 5(3) - 1$. Therefore, $f(3) = 14$.

Matrices

Basics of 2 × 2 Matrices

Addition: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$

Subtraction: Same as addition, except subtract the numbers rather than adding.

Scalar Multiplication: $k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$

Multiplication of Matrices: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$

Coordinate Geometry

This section contains problems dealing with the (x, y) coordinate plane and number lines. Included are slope, distance, midpoint, and conics.

Slope

The formula for finding slope, given two points, (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

The equation of a line is often written in slope-intercept form which is $y = mx + b$, where m is the slope and b is the y-intercept.

Important Information about Slope

- A line that rises to the right has a positive slope and a line that falls to the right has a negative slope.
- A horizontal line has a slope of 0 and a vertical line does not have a slope at all—it is undefined.
- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

Distance

The distance between two points can be found using the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint

The midpoint of two points can be found by taking the average of the x values and the average of the y values.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Conics

Circles, ellipses, parabolas, and hyperbolas are conic sections. The following are the equations for each conic section.

Circle:	$(x - h)^2 + (y - k)^2 = r^2$	where (h, k) is the center and r is the radius.
Ellipse:	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	where (h, k) is the center. If the larger denominator is under y, the y-axis is the major axis. If the larger denominator is under the x-axis, the x-axis is the major axis.
Parabola	$y - k = a(x - h)^2$ or $x - h = a(y - k)^2$	The vertex is (h, k). Parabolas of the first form open up or down. Parabolas of the second form open left or right.
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	

Plane Geometry

Plane geometry covers relationships and properties of plane figures such as triangles, rectangles, circles, trapezoids, and parallelograms. Angle relations, line relations, proof techniques, volume and surface area, and translations, rotations, and reflections are all covered in this section.

To begin this section, it is helpful to become familiar with the vocabulary used in geometry. The list below defines some of the main geometrical terms:

Arc	part of a circumference
Area	the space inside a 2 dimensional figure
Bisect	to cut in 2 equal parts
Circumference	the distance around a circle
Chord	a line segment that goes through a circle, with its endpoint on the circle
Diameter	a chord that goes directly through the center of a circle—the longest line you can draw in a circle
Equidistant	exactly in the middle

Hypotenuse	the longest leg of a right triangle, always opposite the right angle
Parallel	lines in the same plane that will never intersect
Perimeter	the distance around a figure
Perpendicular	2 lines that intersect to form 90-degree angles
Quadrilateral	any four-sided figure
Radius	a line from the center of a circle to a point on the circle (half of the diameter)
Volume	the space inside a 3-dimensional figure

BASIC FORMULAS

Perimeter	the sum of all the sides of a figure
Area of a rectangle	$A = bh$
Area of a triangle	$A = \frac{bh}{2}$
Area of a parallelogram	$A = bh$
Area of a circle	$A = \pi r^2$
Volume of a rectangular solid	$V = lwh$

BASIC GEOMETRIC FACTS

The sum of the angles in a triangle is 180°.

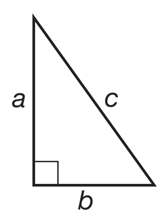
A circle has a total of 360°.

PYTHAGOREAN THEOREM

The **Pythagorean theorem** is an important tool for working with right triangles.

It states: $a^2 + b^2 = c^2$, where a and b represent the legs and c represents the hypotenuse.

This theorem allows you to find the length of any side as along as you know the measure of the other two. So, if leg a = 1 and leg b = 2 in the triangle below, you can find the measure of leg c.



$a^2 + b^2 = c^2$

$1^2 + 2^2 = c^2$

$1 + 4 = c^2$

$5 = c^2$

$\sqrt{5} = c$

Pythagorean Triples

In a **Pythagorean triple**, the square of the largest number equals the sum of the squares of the other two numbers.

Example

As demonstrated: $1^2 + 2^2 = (\sqrt{5})^2$

1, 2, and $\sqrt{5}$ are also a Pythagorean triple because:

$1^2 + 2^2 = 1 + 4 = 5$ and $(\sqrt{5})^2 = 5$.

Pythagorean triples are useful for helping you identify right triangles. Some common Pythagorean triples are:

3:4:5 8:15:17 5:12:13

Multiples of Pythagorean Triples

Any multiple of a Pythagorean triple is also a Pythagorean triple. Therefore, if given 3:4:5, then 9:12:15 is also a Pythagorean triple.

Example

If given a right triangle with sides measuring 6, x, and 10, what is the value of x?

Solution

Because it is a right triangle, use the Pythagorean theorem. Therefore,

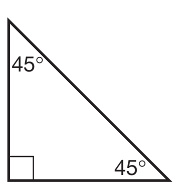
$10^2 - 6^2 = x^2$

$100 - 36 = x^2$

$64 = x^2$

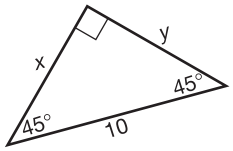
$8 = x$

45-45-90 Right Triangles



A right triangle with two angles each measuring 45 degrees is called an **isosceles right triangle**. In an isosceles right triangle:

- The length of the hypotenuse is $\sqrt{2}$ multiplied by the length of one of the legs of the triangle.
- The length of each leg is $\frac{\sqrt{2}}{2}$ multiplied by the length of the hypotenuse.

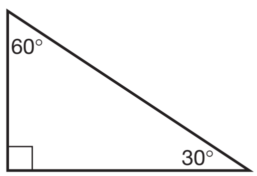


$x = y = \frac{\sqrt{2}}{2} \times \frac{10}{1} = 10\frac{\sqrt{2}}{2} = 5\sqrt{2}$

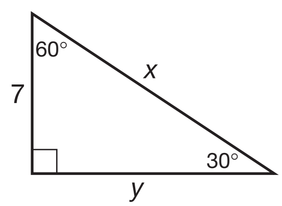
30-60-90 Triangles

In a right triangle with the other angles measuring 30 and 60 degrees:

- The leg opposite the 30-degree angle is half of the length of the hypotenuse. (And, therefore, the hypotenuse is two times the length of the leg opposite the 30-degree angle.)
- The leg opposite the 60-degree angle is $\sqrt{3}$ times the length of the other leg.



Example



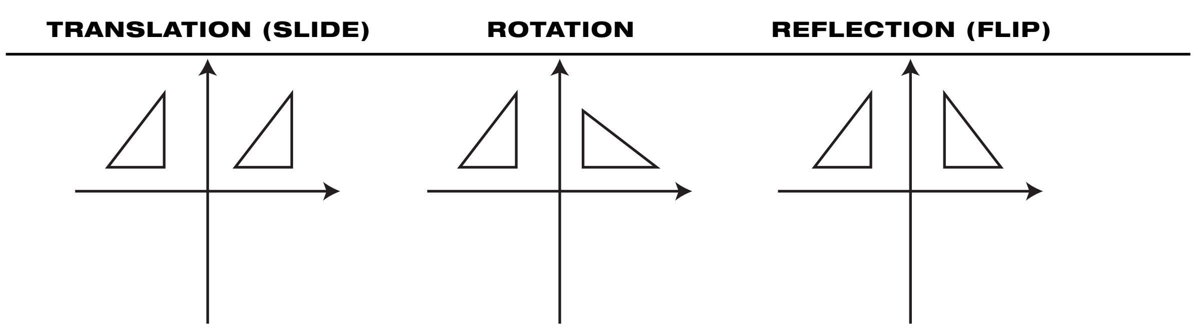
$x = 2 \cdot 7 = 14$ and $y = 7\sqrt{3}$

Congruent

Two figures are congruent if they have the same size and shape.

Translations, rotations, and reflections

Congruent figures can be made to coincide (place one right on top of the other), by using one of the following basic movements.



Trigonometry

Basic trigonometric ratios, graphs, identities, and equations are covered in this section.

Basic Trigonometric Ratios

$\sin A = \frac{opposite}{hypotenuse}$ *opposite* refers to the length of the leg opposite angle A.

$\cos A = \frac{adjacent}{hypotenuse}$ *adjacent* refers to the length of the leg adjacent to angle A.

$\tan A = \frac{opposite}{adjacent}$

Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$

$\tan x = \frac{\sin x}{\cos x}$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

► Glossary of Math Terms

This glossary is a tool to prepare you for the ACT Math Test. You will not be asked any vocabulary questions on the ACT Math Test, so there is no need to memorize any of these terms or definitions. However, reading through this list will familiarize you with general math words and concepts, as well as terms you may encounter in the practice questions. These terms come from all the areas of math found on the ACT, but it is not guaranteed that any of the terms below will be included on an official ACT Math Test.

Base—A number used as a repeated factor in an exponential expression. In 8^5, 8 is the base number.
Base 10—see *Decimal numbers.*
Binary System—One of the simplest numbering systems. The base of the binary system is 2, which means that only the digits 0 and 1 can appear in a binary representation of any number.
Circumference—The distance around the outside of a circle.
Composite number—Any integer that can be divided evenly by a number other than itself and 1. All numbers are either prime or composite.
Counting numbers—Include all whole numbers, with the exception of 0.
Decimal—A number in the base 10 number system. Each place value in a decimal number is worth ten times the place value of the digit to its right.
Denominator—The bottom number in a fraction. The denominator of $\frac{1}{2}$ is 2.
Diameter—A chord which passes through the center of the circle and has endpoints on the circle.
Difference—The result of subtracting one number from another.
Divisible by—Capable of being evenly divided by a given number, without a remainder.
Dividend—The number in a division problem that is being divided. In $32 \div 4 = 8$, 32 is the dividend.
Even number—A counting number that is divisible by 2.
Expanded notation—A method of writing numbers as the sum of their units (hundreds, tens, ones, etc.). The expanded notation for 378 is $300 + 70 + 8$.
Exponent—A number that indicates an operation of repeated multiplication. For instance, 3^4 indicates that the number 3 should be multiplied by itself 4 times.
Factor—One of two or more numbers or variables that are being multiplied together.
Fractal—A geometric figure that is self-similar; that is, any smaller piece of the figure will have roughly the same shape as the whole.
Improper fraction—A fraction whose numerator is the same size as or larger than its denominator. Improper fractions are equal to or greater than 1.
Integer—All of the *whole numbers* and negatives too. Examples are –3, –2, –1, 0, 1, 2, and 3. Note that integers *do not* include fractions, or decimals.
Multiple of—A multiple of a number has that number as one of its factors. 35 is a multiple of 7; it is also a multiple of 5.
Negative number—A real number whose value is less than zero.
Numerator—The top number in a fraction. The numerator of $\frac{1}{4}$ is 1.

Odd number—A counting number that is not divisible by 2.

Percent—A ratio or fraction whose denominator is assumed to be 100, expressed using the percent sign; 98% is equal to $\frac{98}{100}$.

Perimeter—The distance around the outside of a polygon.

Polygon—A closed two-dimensional shape made up of several line segments that are joined together.

Positive number—A real number whose value is greater than zero.

Prime number—A real number that is divisible by only 2 positive factors: 1 and itself.

Product—The result when two numbers are multiplied together.

Proper fraction—A fraction whose denominator is larger than its numerator. Proper fractions are equal to less than 1.

Proportion—A relationship between two equivalent sets of fractions in the form $\frac{a}{b} = \frac{c}{d}$.

Quotient—The result when one number is divided into another.

Radical—The symbol used to signify a root operation.

Radius—Any line segment from the center of the circle to a point on the circle. The radius of a circle is equal to half its diameter.

Ratio—The relationship between two things, expressed as a proportion.

Real numbers—Include fractions and decimals in addition to *integers*.

Reciprocal—One of two numbers which, when multiplied together, give a product of 1. For instance, since $\frac{3}{2} \times \frac{2}{3}$ is equal to 1, $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$.

Remainder—The amount left over after a division problem using whole numbers. Divisible numbers always have a remainder of zero.

Root (square root)—One of two (or more) equal factors of a number. The square root of 36 is 6, because $6 \times 6 = 36$. The cube root of 27 is 3 because $3 \times 3 \times 3 = 27$.

Simplify terms—To combine like terms and reduce an equation to its most basic form.

Variable—A letter, often *x*, used to represent an unknown number value in a problem.

Whole numbers—0, 1, 2, 3, and so on. They do not include negatives, fractions, or decimals.

Appendix: Additional ACT Resources

This book has given you a good start on studying for the ACT exam. However, as you will find in your future courses, one book is seldom enough. It's best to be equipped with several sources, some general, some more specific.

English

Azar, Betty. *Basic English Grammar* (Upper Saddle River, NJ: Prentice Hall, 1998).

LearningExpress. *501 Grammar & Writing Questions* (New York: LearningExpress, 2000).

LearningExpress. *501 Word Analogy Questions* (New York: LearningExpress, 2002).

LearningExpress. *501 Synonym & Antonym Questions* (New York: LearningExpress, 2002).

LearningExpress. *1001 Vocabulary & Spelling Questions* (New York: LearningExpress, 2000).

Lewis, Norman. *Thirty Days to Better English* (New York: New American Library, 1991).

Meyers, Judith N. *Vocabulary & Spelling Success: In 20 Minutes a Day, 3rd Edition* (New York: LearningExpress, 2002).

Princeton Review. *Grammar Smart* (New York: Princeton Review, 2001).

Robinson, Adam. *Word Smart: Building an Educated Vocabulary* (New York: Princeton Review, 2001).

Strunk, William and White E.B. *The Elements of Style, 4th Edition* (Boston: Allyn & Bacon, 2000).

The Ultimate Verbal and Vocabulary Builder for the SAT, ACT, GRE, GMAT, and LSAT (Austin: Lighthouse Review, 1998).

▶ Reading

Blachowicz, Camille and Ogle, Donna. *Reading Comprehension* (New York: Guilford Publications, 2001).

Boone, Robert S. *What You Need to Know About Developing Your Test-Taking Skills: Reading Comprehension* (New York: NTC/Contemporary, 1995).

Chesla, Elizabeth. *Read Better, Remember More, 2nd Edition* (New York: LearningExpress, 2000).

Chesla, Elizabeth. *Reading Comprehension in 20 Minutes a Day, 2nd Edition* (New York: LearningExpress, 2001).

Herrell, Adrienne L. and Jordan, Michael. *Fifty Active Learning Strategies for Improving Reading Comprehension* (Upper Saddle River, NJ: Prentice Hall, 2001).

Hoyt, Linda. *Revisit, Reflect, Retell: Strategies for Improving Reading Comprehension* (Portsmouth, NH: Heinemann, 1998).

LearningExpress. *501 Grammar and Writing Questions, 2nd Edition* (New York: LearningExpress, 2002).

LearningExpress. *501 Reading Comprehension Questions, 2nd Edition* (New York: LearningExpress, 2001).

▶ Math

LearningExpress. *501 Algebra Questions* (New York: LearningExpress, 2002).

LearningExpress. *501 Geometry Questions* (New York: LearningExpress, 2002).

Lerner, Marcia. *Math Smart* (New York: Princeton Review, 2001).

Tarbell, Shirley. *1001 Math Problems*. (New York: LearningExpress, 1999).

Weinfeld, Mark. *ACT Assessment Math Flash 2002* (Stamford: Thomson, 2001).

Weinfeld, Mark. *ACT Math Flash: Proven Techniques for Building Math Power for the ACT* (Stamford: Thomson, 2000).

▶ Science

Giere, Ronald N. *Understanding Scientific Reasoning, 2nd Edition* (Austin: Holt, 1998).

▶ Other ACT Study Guides

ACT Assessment Success 2003 (New York: Petersons, 2002).

Bobrow, Jerry et. al. *Cliffs TestPrep ACT Preparation Guide* (Hoboken: Wiley, 2000).

Domzalski, Shawn Michael. *Crash Course for the ACT: The Last-Minute Guide to Scoring High* (New York: Princeton Review, 2000).

Ehrenhaft, George et. al. *How to Prepare for the ACT* (Hauppauge, NY: Barron's, 2001).

Getting into the ACT: Official Guide to the ACT Assessment (New York: HBJ, 1997).
Kaplan ACT 2000 with CD-ROM (New York: Kaplan, 2002).
Magliore, Kim and Silver, Theodore. *Cracking the ACT* (New York: Princeton Review, 2002).
Panic Plan for the ACT (New York: Petersons, 2000).

▶ Study Guides

Fry, Ronald. *Ace Any Test* (Franklin Lake, NJ: Career Press, 1996).
Huntley, Sara Beth and Smethurst, Wood. *Study Power Workbook: Exercises in Study Skills to Improve Your Learning and Your Grades* (Cambridge: Brookline Books, 1999).
Luckie, William R., and Smethurst, Wood. *Study Power: Study Skills to Improve YourLearning and Your Grades* (Cambridge: Brookline Books, 1997).
Meyers, Judith. *The Secrets of Taking Any Test, 2nd Edition* (New York: LearningExpress, 2000).
Wood, Gail. *How to Study, 2nd Edition* (New York: LearningExpress, 2000).
Semones, James. *Effective Study Skills: A Step-by-Step System for Achieving Student Success* (Washington, DC: Thomson, 1991).

▶ Websites

www.act.org— The official ACT site.
www.testprep.compracticehdr.shtml—Provides practice tests for the ACT exam.
www.powerprep.com—Provides strategies, tutoring, software, diagnostic and online practice tests for the ACT exam.
www.review.com—Provides tutoring and test preparation for the ACT exam.
www.kaplan.com—Provides tutoring, test preparation, and general information for the ACT exam.
www.act-sat-prep.com—Provides practice exams and strategies for taking the ACT exam.